# The dissipation subrange in wind wave spectra

*By* S. A. KITAIGORODSKII

Matematisk-fysiske Meddelelser **42**:5

Det Kongelige Danske Videnskabernes Selskab
*The Royal Danish Academy of Sciences and Letters*

Commissioner: Munksgaard · Copenhagen 1992

# Contents



## *Abstract*

In Kitaigorodskii (1983) it was suggested that due to the breaking of wind waves in deep water the dissipation of wave energy is restricted to a range of wave numbers much higher than the wave numbers typical for the *so called equilibrium range*, or much higher than the peak wave number $k_p$ and peak frequency $\omega_p$. This prediction, similar to the prediction of the existence of dissipation subrange in Kolmogoroff's three dimensional tubulence never before was properly verified for the obvious reasons: difficulties related to the measurements and interpretation of the random wave field properties at very high frequencies (or wave numbers). In this paper an attempt is made to summarize the results of recent field experiments (Leykin and Rozenberg, 1984, Tang and Shemdin, 1983, Birch and Ewing, 1986. Hansen et al., 1990, Banner et al., 1989, Banner, 1990 and some others) with the purpose to demonstrate that the *rapid spectral fall off* needed for determination of the boundaries of dissipation subrange in wave number frequence domain, seems to be an intrinsic property of rather *well-developed seas*.

It is shown that in dissipation subrange the spectrum most likely has the form $S(\omega) = \beta g^2 \omega^{-5}$, $\beta \simeq 0.025$, g-gravity, and dissipation of energy is restricted to a range of frequencies $\omega > \omega_g$ much higher than the frequencies of the dominant waves. The characteristics of the *transition* from rear high frequency parts and high wave number parts of wave spectra to the *dissipation* subrange are summarized, and the contradictions between different interpretation of wind speed dependence of radar back scatter measurements are also discussed.

S. A. KITAIGORODSKII
Department of Earth & Planetary Sciences
The Johns Hopkins University
Baltimore, Maryland 21218


Printed in Denmark by Special-Trykkeriet Viborg a-s
ISSN 0023-3323 ISBN 87-7304-230-7

# 1. Introduction

Since the 1981 symposium on wave dynamics and radio probing of the ocean surface in Miami (proceedings were published in 1986 by Plenum Press) many interesting publications about the equilibrium spectra of wind waves appeared in oceanographic literature. Beginning with the work's of Kitaigorodskii (1983) and Philips (1985), a great deal of attention was devoted to the explanation of the wind speed dependence of the rear face of the spectra of surface gravity waves, both by using either concepts of statistical equilibrium of the Kolmogoroff's type in weakly nonlinear surface gravity wave field (Kitaigorodskii, 1983, Zakharov and Zaslavskii, 1982), or alternative model of the statistical equilibrium based on the balance of *source terms* (Phillips, 1985, Komen et al., 1984). The forms of the equilibrium spectra in these two models are not too different from each other (Kitaigorodskii, 1987), which make it difficult to *distinguish* between the types of statistical equilibrium only on the basis of the information about the *rear faces* of the frequency and wave number spectra. This becomes even more evident, after recent work by Banner (1990), who put special attention to the probably underestimated before the *important role of k-dependent* type of angular distribution of wave energy propagation in shaping rear faces of frequency wave spectras not far from its peak. This viewpoint was just briefly mentioned in Kitaigorodskii et al. (1975) (see footnotes on p. 114 in this paper), whereas Banner (1990) had attempted to establish *empirically* the canonical form of wave spectra in the *whole energy containing region* of the two-dimensional wave spectra $\psi(\underset{\sim}{k}) = \psi(k,\theta)$, ($\underset{\sim}{k} = (k \cos \theta, k \sin \theta)$.

Contrary to *asymptotic* arguments of *statistical* equilibrium in weakly nonlinear field of surface gravity waves, which leads to *wind* dependence of $\psi(k,\theta)$ through the dependence of *energy* and *action* fluxes from wind speed (Kitaigorodskii, 1983, 1987), Banner (1990) formulated 2-D wave number spectral model using empirical form of *directional* frequency spectra of Donelan et al. (1985), with *extrapolation* of their $\omega < 3\,\omega_p$, $\omega_p$-peak frequency to much higher wave numbers. Banner (1990) argues that the $k^{-4}$ form of the rear side of the $\psi(k,\theta)$ above the peak enhancement region is in agreement with latest observations, and that the *broad* directional distribution (independent of $k/k_p$ values, $k_p$-peak wave number) occurs approximately only at $k/k_p > 10$. For smaller $k/k_p$ Banner's (1990) model was able to demonstrate that the *prescribed* spreading function according to Donelan et al. (1985), can easily explain two *observed* features in *frequency* spectra of ocean gravity waves, i.e. wind dependence in the region close to the peak ($\omega/\omega_p \leq 3$) and *transition* from $\omega^{-4}$ to $\omega^{-5}$ form, *whose typical frequency noticeably varies with* $\omega_p$ (!). However Banner's (1990) *model* deals with relatively *low wave numbers* and *does not* consider high wave number tail ($k/k_p > 10$), where dissipation effects can be of primary importance according to Kitaigorodskii (1983) description of the *dissipation* subrange. The latter approach has received some support in a recent paper by Hansen et al. (1990), where transition to $\omega^{-5}$ form in the frequency spectra was found to be consistent with transition to $k^{-4}$ in *high wave number* region of $\psi(\underline{k})$. The transition from *wind dependent* $k^{-7/2}$ form of spatial spectrum $\psi(\underline{k})$ to $k^{-4}$ form, which according to Kitaigorodskii (1983) indicates the dissipation subrange, appears in the data of SWOP experiments (cf. recalculations of SWOP data in Kitaigorodskii (1984). To my knowledge this was long time the only *dirtect* evidence of occurrence of the more rapid spectral falloff in high wave number tail in wave number spectra. Of course the data which contains $\psi(k,\theta)$ spectra satisfying $k^{-4}$ form are much more numerous (Phillips, 1977). The latest among them seems to be Banner et al. (1989), recent stereophoto measurements of $k^{-4}$ form in the range of short wavelengths 0,2-1.6 m which we *will analyze later*. The very question about the existence of the *dissipation subrange* is far from being only of academic interest – the radar backscattering wind dependence and wave number dependence can be explained only by knowing the behaviour of high frequency and high wave number tails of the spectrum (see Wu, 1990). Data, used by Wu (1990), seems to indicate rather clearly that radar returns wind dependence is different for different wavelengths: for L and $L_p$ bands (and lower wavelength) returns can be explained using the wind-dependent statistical equilibrium (Kitaigorodskii, 1983, Phillips, 1985), whereas the X and C bands (and shorter wavelengths) cannot be explained without the existence of *dissipation subrange* with it's practically k-independent contribution to the scattering cross section in radar return proportional to the slope spectra of gravity waves (Wu, 1990). Indeed the dissipation subrange according to radar returns must be observed (or exist) at wave numbers much higher than those which were originally found by Phillips in 1958 (Phillips, 1958) and Phillips (1966). For example the data of Guinard et al. (1971), analysed recently by Wu (1990) in respect to the variations of the radar return with surface wave number, seems to locate the dissipation subrange in the range $12\text{cm} > \lambda > 1{,}25\text{cm}$. This appears to be a rather extreme conclusion; for example, the results of recent stereophotography indicating a $k^{-4}$ form with broad angular distribution in the range of wave lengths 20-150cm, *can be considered* also as an evidence of dissipation subrange. Wu (1990) analysis seems to be rather in accord with Phillips (1987) assertion that the equilibrium range prevails for gravity wave components, where *dissipation* subrange exist only for *shortest gravity and gravity-capillary* wave components. Note also that in situ measurements used by Kitaigorodskii (1983) and Phillips (1985) were limited to components no shorter than say 1-1.5m. Thus according to these authors, as well as Banner et al. (1989) it seems that the typical value of upper bound of dissipation subrange is 1-1.5 meter (!). However it must not be forgotten also that according to Kitaigorodskii (1983) analysis and Hansen et al. (1990) the typical transitional wave number $k_g$ and frequency $\omega_g$ for the "beginning" of dissipation subrange in wave spectra depend both on wind speed and the stage of development (decrease of $k_g$ and $\omega_g$ with wind speed, as well as with *fetch* or *duration* (cf. Fig. 5 in Hansen et al. (1990)). Thus we can expect that for young waves and not high wind speeds the dissipation "subrange" is moving to the "miscroscales" of shortest gravity ripples and capillary gravity waves, where for moderate and strong winds and rather well developed waves dissipation "subrange" can be observed in more wide ranges of wave numbers and frequencies higher than the peak values, but lower than the scales of gravity capillary ripples. Still the available information about transitional frequency $\omega_g$ according to Forristal (1981), Kahma (1981), Kitaigorodskii (1983), Hansen et al. (1990), doesn't show a big variation in $\omega_g$, whose typical value looks close to $4^g/U_a(U_a$ – wind speed). The very fact that the dissipation subrange is characterized by power exponent 4, give rise to the attempts to characterized the surface geometry in *equilibrium range*, (with smaller exponents), by the use of fractal dimensions which in such case becomes both relevant and useful (Glazman, 1988). It is interesting that the rapid spectral falloff needed for determination of the boundary of dissipation subrange seems to be an intrinsic property of relatively well developed sea. So according to fractal description of the sea surface the surface microscale $h \approx 1$m (Kolmogroff's microscale) is in good agreement with the values of transitional wave number $k_g$ and frequency $\omega_g$ including those which appear *indirectly* in the Phillips (1985) and Banner et al. (1989) papers. Their results we'll reanalyze, but it seems from above that it is instructive to establish the cortrespondence between Kitaigorodskii (1983) "transitional" scales and fractal model of the sea with its inner microscale h. We'll not discuss this topic here, but instead we'll turn our attention to the proof of very existence of the dissipation subrange in the experimental data (both old and recent), but especially those which were not available at time when our review paper (Kitaigorodskii, 1986) was written.

## 2. Statistical equilibrium, saturation and dissipation subrange in wind wave spectra

2.1. The rather complete description of the commonly used spectral characteristics of wave field which are either measurable or calculable can be found in the papers of Kitaigorodskii (1987) and Banner (1990). Both contain a good account of the spectral description of random wind wave field. We briefly repeat here what in this respect will be needed for further discussions.

The Fourier series representation of the surface $\zeta(\underset{\sim}{x},t)$.

$$\zeta(\underset{\sim}{x}, t) = (\iint \exp \{i (\underset{\sim}{k}, \underset{\sim}{x} - \omega t) dZ_\zeta (k,\omega) = \int d \underset{\sim}{k} \int d\omega \zeta_{\underset{\sim}{k}\omega} \exp \{i (\underset{\sim}{k} x - \omega t)\} \tag{1}$$

$\{\underset{\sim}{k} = (k_1,k_2) = (k \cos \phi, k \sin \phi)$ is wave number vector, $\omega$ is frequency$\}$ is often used in the description of random wave field (in this case $dZ_\zeta(\underset{\sim}{k},\omega)$ is Fourier-Stilties amplitude).

The *symmetrical* wave spectrum $E_s(\underset{\sim}{k},\omega)$ defined as

$$E_s (\underset{\sim}{k}, \omega) = E_s (-\underset{\sim}{k}. -\omega) = \left\langle \zeta_{\underset{\sim}{k}\omega} \zeta^*_{\underset{\sim}{k}'\omega'} \right\rangle \tag{2}$$

is the Fourier transform of the covariance $B(\underset{\sim}{r},\tau)$

$$E_s (\underset{\sim}{k}, \omega) = (2\pi)^{-3} \int d\underset{\sim}{r} \int d\tau\, B(\underset{\sim}{r},\tau) \{ -i (\underset{\sim}{k}\underset{\sim}{r} - \omega\tau)\} \tag{3}$$

with normalization condition

$$\left\langle \zeta^2 \right\rangle = B (0,0) = \int d\underset{\sim}{k} \int d\omega\, E_s (\underset{\sim}{k},\omega) \tag{4}$$

and

$$B (\underset{\sim}{r},\tau) = B (\underset{\sim}{x} +\underset{\sim}{r}, \tau, \underset{\sim}{x}, t) \tag{5}$$

The reduced *symmetrical* (measurable) wave number spectres $\psi_s(\underset{\sim}{k})$ and frequency spectra $S(\omega)$ can be obtained by intergration over $\omega$ and over k

$$\psi_s (\underset{\sim}{k}) = \int d\omega\, E_s (k,\omega) \tag{6}$$

$$S_s (\omega) = \int d\underset{\sim}{k}\, E_s (k,\omega) \tag{7}$$

$\psi_s(\underset{\sim}{k})$arises from frozen spatial image analysis and does not contain actual wave propagation information partitioning the wave energy equally to components 180° apart.

Note, (Kitaigorodskii, 1986) that

$$\Psi_s (\underset{\sim}{k}) = \frac{1}{2} [F (\underset{\sim}{k}) + F (-\underset{\sim}{k})] \tag{8}$$

where the directional wave number spectrum $F(\underset{\sim}{k})$ is defined as

$$\frac{1}{2} F (\underset{\sim}{k})\, \delta (\underset{\sim}{k} - \underset{\sim}{k}') = \left\langle \eta^+_{\underset{\sim}{k}} (\eta^+_{k'})^* \right\rangle \tag{9}$$

and $F(-\underset{\sim}{k})$ as

$$\frac{1}{2} F (-\underset{\sim}{k})\, \delta (\underset{\sim}{k} - \underset{\sim}{k}') = \left\langle \eta^-_{k} (\eta^-_{k'})^* \right\rangle \tag{10}$$

where in (1)

$$\zeta_{\underset{\sim}{k}\omega} = \eta^+_k\, \delta (\omega - \delta) + \eta^-_k\, \delta (\omega + \delta) \tag{11}$$

and random coefficients $\eta_k^+, \eta_k^-$ are the amplitudes of free liear surface gravity waves propagating in the positive and negative direction of the vector $\underset{\sim}{k}$, and $\sigma$ satisfies the dispersion relationship for surface waves. That is why for a weakly nonlinear wave field the Fourier series representation (1) is a more natural tool for theoretical analysis than in studies of turbulent random fields. The directional wave number spectrum $F(\underset{\sim}{k})$

$$F(\underset{\sim}{k}) = 2 \int_0^\infty E_s(\underset{\sim}{k}, \omega)\, d\omega \tag{12}$$

represents the actual wave number distribution of wave energy propagation.

Among the often measurable spatial characteristics it is worthwhile to mention also *one dimensional* (transverse) spectra

$$\phi_s(k_1) = \int_{-\infty}^{\infty} \Psi_s(k_1, k_2)\, dk_2 \tag{13}$$

$$\phi_s(k_2) = \int_{-\infty}^{\infty} \Psi_s(k_1, k_2)\, dk_1 \tag{14}$$

and among the calculated reduced spectras the spectrum of wave number moduli, characterizing energy distribution over k regardless of the direction of wave propagation defined as

$$\chi(\underset{\sim}{k}) = \int_{|k|=\kappa} \Psi_s(\underset{\sim}{k})\, d\underset{\sim}{k} = \int_{-\pi}^{+\pi} \Psi_s(k,\theta)\, k d\theta \tag{15}$$

or the spectrum $F_k$ *averaged* over all directions of wave propagation

$$F_k = \int_\sigma F(\underset{\sim}{k})\, d\theta = \int_{-\pi}^{+\pi} F(k,\theta)\, d\theta \tag{16}$$

It is evident from (8-16) that to calculate the *reduced* spatial spectras (13, 16) we need either a model for directional wave number spectras $F(\underset{\sim}{k})$, or the empirical description of the whole 2-dimensional symmetrical spectra $\psi_s(\underset{\sim}{k})$.

Now we'll try to introduce the noncontroversial definitions of what is considered in the literature as *equilibrium spectra* (or *equilibrium range* in wave spectra), *saturation* (or saturation range in wave spectra), and finally what we'll call *dissipation subrange* in wind wave spectra.

2.2. The evolution of the directional wave number spectrum $F(\underset{\sim}{k}, x, t)$ has been described by the so-called radiative transfer equation

$$\frac{DF(\underset{\sim}{k})}{Dt} = \frac{\partial F(\underset{\sim}{k})}{\partial t} + c_g \nabla F(\underset{\sim}{k}) = S_{in}(\underset{\sim}{k}) + S_{nl}(\underset{\sim}{k}) + S_{diss}(\underset{\sim}{k}) \tag{17}$$

Here $S_{in}(\underset{\sim}{k})$, $S_{nl}$ $(\underset{\sim}{k})$ and $S_{diss}$ $(\underset{\sim}{k})$ are the *source terms*, representing the spectral distributions of wind input, nonlinear interactions with other wave components and dissipation through wave breaking and wave turbulence interactions (Kitaigorodskii, Lumley 1983). If for certain ranges of $\underset{\sim}{k}(k,\theta)$

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \bar{C}_g \nabla F = 0 \tag{18}$$

such a region of wave numbers can be called an *equilibrium* range of F(k), and corresponding form of $F(\underset{\sim}{k})$ spectra in this region equilibrium spectra. In practice we of course met with the situation when in certain part of $\underset{\sim}{k}$ *domain*

$$\left|\frac{DF}{Dt}\right| << S, \text{ where } S = S_{ne} + S_{in} + S_{diss} \tag{19}$$

which means that this part of the spectrum is in quasy-equilibrium. The condition (19) seems to be *fulfilled* in fetch grwoth situation for *major parts* of rear faces of wind wave spectra, and that is why the similarity descriptions of wind wave development according to Kitaigorodskii scaling (Kitaigorodskii, 1960, and Pierson and Moscowitz, 1964) and description *of parametric wave spectra according to* Haselmann et al. (1976) are both applicable and successful in the predictions of the growth of wind wave field with fetch.

2.3. The most general analysis of equilibrium spectra of the type (18, 19) was done recently by Phillips (1985), who used the dimensional arguments in description of $S_{diss}$ and assumption about the equality by the order of magnitude of all source terms in (17). Later on, Banner (1990) prefer to rely on the pure *empirically* chosen canonial form of 2-D $\psi_s(k)$ in the *equilibrium* range of $\psi_s\underset{\sim}{k}$ (19), disagreeing partially both with Kitaigorodskii (1983) and Phillips (1985) descriptions of equilibrium conditions, and putting special attention to the role of k-dependent form of angular destribution of energy in *equilibrium part of* $\psi_s(\underset{\sim}{k})$. The fact that we must talk about an equilibrium not only with respect to k, but also to θ, i.e. above certain region k,θ in the spatial spectrum $\psi_s(\underset{\sim}{k})$ was first pointet out in Kitaigorodskii et al (1975) in their discussions of the Phillips (1958) hypothesis about the special *type of equilibrium*, produced by limitation on the growth of wave spectral components imposed by gravitational instability – surface wave breaking.

The form of the spectra predicted by Phillips (1958) on the basis of this idea

$$\psi_s(k,\theta) = Bk^{-4} \zeta(\theta) \tag{20}$$

was later on called a *saturation form*, and the corresponding range of (k,θ) saturation *range* of wind-wave spectra or *saturation spectra*. As the boundaries of such range in k,θ plane are not known a priori, no unambiguous conclusions about the shape of the $\psi(k_1)$, $\psi k_2)$ or $\chi(k)$ spectra can be drawn even if the function $\varsigma(\theta)$ in (20) (satisfing the standard normalization condition $\int_{\infty}^{\infty}\varsigma(\theta)d\,\theta = 1$) is known, unless one makes some additional assumptions not following from the similarity theory itself, such as for instance one utilized by Phillips (1966) who assumed that in (20)

$$\zeta(\theta) = \begin{cases} \frac{1}{\pi} \text{ at } \theta \le \theta_m = \frac{\pi}{2} \\ 0 \text{ at } \theta > \theta_m \end{cases} \qquad (21)$$

where $\theta_m$ corresponds to the dominant wave direction. However to some extent the general form of formulae (20) contradicts the data of the most detailed investigation of angular energy distribution in the wave spectrum, as obtained by Longnet-Higgins et al. (1963), Ewing (1969) and most recently in a comprehensive study of Donelan et al. (1985). It seems that k-independence of angular distribution and the lack of sensitivity to both wind strength and wind direction (tendency to isotropy), as in (20, 21), can be observed only for shrotest wave components (see for example Banner et al. 1989). Because of this both in Kitaigorodskii et al. (1975) and Kitaigorodskii (1983), the physical hypothesis about the saturation or equilibrium spectra was formulated *directly* to the statistical characteristics of wave field, already averaged over all directions of wave propagation, i.e., directly to the spectra $\chi(k)$ or $F(k)$ as it is done often in the theory of small scale axsymmetric turbulence. One of the reasons why k-independent type of angular distribution, including isotropy, is of particular interest in deriving the average (over all angles) wave statistics is due to the fact that the *very* special type of statistical equilibrium in weakly nonlinear field of surface gravity waves can be described by particular form of equation (17)

$$S_{nl}(\underset{\sim}{k}) \equiv 0 \qquad (22)$$

*which is known as wave kinetic equation.*

2.4. The forms of equilibrium spectra corresponding to (22) for statistically averaged characteristics was first derived by Kitaigorodskii (1983). For *isotropic* wave field the exact analytical solution of (22) was studied by Zakharoff and Filonenko (1964). Later on in the series of papers by Zakharoff and Zaslavskii (1982, 1983) it was shown that (22) leads to two special forms of Kolmogoroff's type *cascade* spectra $F_k$ (Kitaigorodskii, 1987) and one of them based on *action flux towards low wave numbers* were successfully applied for the description and parametrization of wave field in the case of the so called *fully developed waves.* We'll *not consider* the fully developed wind wave spectra as an example of *equilibrium spectra* leaving the latter name just for range of wave numbers (and frequencies) at least larger than the peak wave number $k_p$ (or frequency $\omega_p$).

Finally let us introduce like in Kitaigorodskii (1983) and Hansen et al. (1990) the definition what can be called – the *dissipation subrange* in wind wave spectra. According to these authors this is the range of wave numbers $k \geqslant \hat{k}_g$ where

$$S_{nl}(\underset{\sim}{k}) = 0 \tag{23}$$

and

$$S_{nl}(\underset{\sim}{k}) - S_{diss}(\underset{\sim}{k}) = 0 \quad \text{for } k \geq \hat{k}_g \tag{24}$$

Here it is assumed that the wind energy input is negligible near the transitional wave number $\hat{k}_g$ and that wave breaking is important only at wave numbers higher than the wave number $\hat{k}_g$ of gravitational instability which supposedly is *much higher* than $k_p$, and can depend also on direction $\theta$. The basic role of weak nonlinear interactions is then in redistributing energy from the range of $k_g \leqslant k \leqslant \hat{k}_g$ to new waves with $k \leqslant k_p$ (Kitaigorodskii, 1983, Zaharoff and Zaslavskii, 1982) and to dissipation $k \geqslant \hat{k}_g$ in such a way that nonlinear divergence of energy in the range $k_p \leqslant k \leqslant \hat{k}_g$ is in balance with wind energy input $S_{in}$ in a stationary wave field (Phillips, 1985). Here by $\hat{k}_g$ it is worthwhile to understand the average value $\hat{k}_g = \int k_g(\theta)d\theta$, so that angular distribution can weight towards $\hat{k}_g$ and transitional frequency $\omega_g$ can be related to $\hat{k}_g$ in usual way through isotropic dispersion relationship ($\omega_g = (g\hat{k}_g)^{1/2}$) (if Doppler shifting is not taken into account).

Thus from this definition of the *dissipation subrange* it follows that in wave number (or frequency domain) the subrange have to occur as a more *rapid* spectral fall of compared with one in *equilibrium range of wave spectra, and that without the latter there will be* no dissipation subrange *at all* if we accept the above introduced terminology. According to this terminology $S_{nl}(\underset{\sim}{k}) \simeq S_{inp.}(\underset{\sim}{k})$ in the *equilibrium* range of the spectra, whereas $S_{nl} \simeq S_{in} = S_{diss}$ just a particular type of statistical equilibrium corresponding to the Kolmogoroff's type of theory of the inertial subrange of quasy-isotropic turbulence, where the *dissipation subrange* is introduced in high wave number part of the spectra due to the direct action of molecular viscosity.

2.5. In the wind wave field the first constructive suggestion about high wave number and high frequency part of spectra was made by Phillips (1958). Phillips (1958) idea became well known as hypothesis about the saturation of wave components due to the limitation imposed on their growth by breaking process. For frequency spectra it gives simple and very elegant result

$$S(\omega) = \beta g^2 \omega^{-5} \tag{25}$$

where $\beta$ is nondimensional universal constant.

However only after Kitaigorodskii (1983) suggestion to consider (24, 25) as an asymptotic form of the *disspipation subrange* in the quasy-equilibrium wave spectra, it becomes clear that Phillips (1958) initial argument must be revisited in favor of the existence of an intrinsic "inner scale" of the sea surface on analogy with Kolmogoroff's inner scale in 3-D turbulence which can corespond to the high wave number (high frequency) fall of the equilibrium spectra. However, such inner scales appear in wave data analysis as transitional wave number $k_g$ (or frequency ($\omega_g$), and it becomes customary to consider the deviations from *new equilibrium form* (Kitaigorodskii (1983), Phillips (1985) to be associated with *gravitational instability* (wave breaking), but not viscosity and therefore with transition to the *dissipation subrange* in wave field. In the next two sections I'll discuss the experimental data about spatial and temporal statistical characteristics of wind wave field with *the purpose* to indicate *such transition and interprete it as a noncontroversial evidence of the existence of dissipation subrange* in wind wave spectra.

## 3. Dissipation subrange in wind wave spectra (definitions)

3.a. In 2-D wavenumber space let us first restrict our attention to wave numbers well below those associated with capillary ripples and those directly influenced by viscosity, so that

$$k << k_T = (\rho_\omega g T^{-1/2}) \; ; \; k << k_\nu = g^{1/4} \nu^{-1/2} \tag{26}$$

(T-surface tension, $\nu$ – kinematic viscosity and $p_\omega$ – density of sea water) and also well above those k which are associated with strong direct energy input from wind k = $k_{in} \geqslant k_p$, so that

$$k >> k_p = \omega_p^2 g^{-1} \tag{27}$$

Then we can expect that somewhere in the region (26-27) wind wave field loses its directionality (the function $k_{in} = k_{in}(\theta)$), must have a clear defined maximum at $\theta = \theta_m$, where $\theta_m$ coincides with direction of wind, or direction of propagation of dominant wind waves. We would assume for moment that there must exist the spreading cut off at scales *much shorter* than $k_{in}$ (where the angular distribution approach isotropy).

3.b. According to Banner (1990) the spreading cut off is still unlikely to occur at $k/k_p \leqslant 2.6$ (as was initially suggested in Donelan et al. (1985) and is expected to be at much shorter scales (Banner et al., 1989). Fig. 2 in Banner (1990) places *spreading* cut off approximately at $k/k_p \approx 9$-10. Even though this number cannot be immediately transfered to frequency domain (Doppler shifting), it is *useful to remember* that it *roughly gives* $\omega/\omega_p = 3$ as lower (in frequency) boundary of the region where wave field loses it's *directionality* (at least qualitatively).

3.c. If one follows Zaharoff and Zaslavskii (1982, 1983) applications of the theory of weakly nonlinear surface gravity waves top the analysis of wind wave data then the case of rather well developed waves must correspond to $k_p=(2-4)\ g/U_a^2$, $U_a$-wind-speed, with the corresponding region of energy input equal approximately to

$$k_{inp} \approx (4-6)\frac{g}{U_a^2} = (2-3)\,k_p \tag{28}$$

Thus we can see that in wave number space there is a region

$$(9\text{-}10)\,k_p \geq (2\text{-}3)\,k_p = k_{In} \tag{29}$$

where *inspite of directionality of wave field* the nonlinear interactions can still play a major role both in redistributing energy between different directions as well as giving rise to smaller scale waves. However it must not be forgotten that in this region both the directionality and wave age dependence (parametrically at least) can influence the description of 2-D wave field characteristics. This was demonstrated in Banner (1990) who simply accept Donelan et al. (1985) *empirical model* of wave spectra with their angular distribution extrapolated to *high wave numbers*.

3.d. The region, described in 3.c is characterized by strong directionality and symmetry relative to the direction of propagation of dominant waves (or mean wind direction). However (29) can still be called an *equilibrium range of wind wave spectra* contrary to Phillips (1985) and Banner (1990) models, and similarity hypothesis can be applied here for statistically averaged characteristics of wave field (according to Kitaigorodskii (1983) definition of statistical equilibrium). The boundaries $k_{bound}$ of region where inside this equilibrium range dissipation due to the breaking becomes important can depend noticeably also on angle, i.e.

$$k_{bound} = k_{diss}\,(\theta) \tag{30}$$

and we'll use an effective value $\hat{k}_g$, defined as

$$\hat{k}_g = \int_\theta k_{diss}\,(\theta)\,d\theta \tag{31}$$

Then the transitional frequency $\omega_g$ from the nondissipative part of equilibrium range to the dissipative part can be defined as

$$\omega_g = \sqrt{g\hat{k}_g} \tag{32}$$

Such value of $\omega_g$ (defined through $\hat{k}g$) *must be close*, but not necessarily equal to *experimentally derived* $\omega_g$ *as a rapid transitional frequency* or beginning of rapid spectral fall off on the rear face of *frequency* spectra, associated with asymptotic approach to *saturation* form (see (38) below).

3.e. The general similarity hypothesis applied to the region

$\left.\begin{matrix} k_T \\ k_V \end{matrix}\right\} > k > k_{in}$, must be based on the dependence of statistical characteristics of wind wave field on the values of parameters g, k (or ω) and $\varepsilon_o$, where $\varepsilon_o = \int\varepsilon(\theta)d\theta$ is a constant energy flux from the region of energy input through the nondissipative part of spectra toward high wave numbers. According to this hypothesis we have (Kitaigorodskii, 1983) for such characteristics as energy spectrum $F_k$ and wave action spectral density $N_k = \frac{gFk}{\sigma(k)}$ ($\sigma = \sigma(k)$-isotopic dispersion relationship) the following expressions

$$F_k = \int F(\underset{\sim}{k})\, d\theta = \varepsilon_o^{1/3} g^{1/2} k^{-7/2} \phi_1\left(\frac{\bar{k}}{\bar{k}_g}\right) \tag{33}$$

$$N_k = \int N(\underset{\sim}{k})\, d\theta = \varepsilon_o^{1/3} k^{-4} \phi_2\left(\frac{k}{k_g}\right) \tag{34}$$

In *nondissipative* range of wave spectral characteristics ($k/k_g << 1$), the nonlinear interactions must play a major role, and because they are cubic in wave amplitude it follows (Kitaigorodskii, 1983; Phillips, 1985) that

$$\phi_1 = \phi_2 = A \tag{35}$$

where A is absolute constant, supposedly close to unity. We'll define here the *dissipative subrange* as a region where the governing parameters are those that *determine continuity* of the wave surface and therefore asymptotically $F_k$ and $N_k$ becomes independent on $\varepsilon_o$ so that

$$\phi_1\left(\frac{k}{k_g}\right) = \phi_2\left(\frac{k}{k_g}\right) = B\left(\frac{k}{k_g}\right)^{-1/2} \tag{36}$$

where B is another absolute constant. The *asymptotic prediction* (33, 36) corresponds to Phillips (1958) initial *saturation* form of the spectra. Here it is based on the value

$$\hat{k}_g = C\, g/\varepsilon_o^{2/3} \tag{37}$$

where C is another numerical constant not necessarily of order one, because the dissipative process associated with wave *breaking* is not well defined (both physically and formally). However it must not be forgotten that in 2-D wave number space some of the values of $k_g$ for example $k_g(\theta max)$ where $\theta_{max}$ coincide with direction of dominant waves, can be much smaller than the value $\hat{k}_g$ (37). That is possibly one of the reasons why in Banner (1990) model wave number spectral density *slice* in the dominant wave direction have a clear defined range (36) *practically int he whole region* (29). The effect of the modulation of the spectra (33, 36) by *orbital peak velocities* permits Banner (1990) to get $\omega^{-4}$ form of the frequency spectra close enough to the peak ($^{\omega}/\omega_p < 4$) (see Fig. 7 in Banner (1990)) with the transition to $\omega^{-5}$ form at the values of $\omega_g$ roughly satisfying (32) and equality (Hansen et al., 1990).

$$\omega_g \cong \frac{B}{A}\;\; g/\varepsilon_0^{1/3} \tag{38}$$

We will examine the empirical data and methods of determination of the constants A,B,C, in next section.

# 4. Transition to dissipation subrange (experimental data)

4.a. SWOP spectra. Dealing with spatial characteristics it is natural to start with the classical SWOP data (Cote et al., 1960). McLeish and Ross (1985) when examining the relationships between spatial and frequency spectra for SWOP data have *assumed* that the effect of presence of wind underlying current is evident. According to these authors the SWOP results had peak spectral levels *well below* that of standard fetch limited conditions. However according to recalculations of Kitaigorodskii (1984) the *normalized* spectral densities $F_k$ have a clear defined $k^{-7/2}$ region. In the spectrum $\chi(k)$ a transition to $k^{-4}$ form occur at $\hat{k}_g = 3{,}2k_p$ and $\hat{k}_g = 0{,}2m^{-1}$ (which corresponds to rather big value of $\lambda_g \sim 30m$. The latter value justifies the *neglect* of the effect Doppler shifting by permanent drift currents even as strong as 1m/sec and leads to the value of transitional frequency $\omega_g \simeq 1.78\omega_p$ which is close (but a little bit low) for usually *observed* transition to $\omega^{-5}$ region in frequency spectra at $\omega_g \sim \frac{4g}{Ua}$, (with $k_p \approx 2\div4\,\frac{g}{Ua^2}$ $\omega_g \approx 1.78\ \omega_p \approx 2.5\text{-}3.7\,\frac{g}{Ua}$). Such *low value* of $\omega_g$ can be explained also by the neglect in doppler shifting by *peak frequency orbital* velocities, (Kitaigorodskii et al., 1975) because the equilibrium region of $k^{-7/2}$ in SWOP spectra occur at $k/k_p \geqslant 1.3$ ($\frac{\omega}{\omega p} \geqslant 1.14$) which practically excludes the peak *enhancement* region (in the frequency range $\frac{\omega}{\omega_p} < 1.3$). The latter fact leads Banner (1990) to conclusion that SWOP spectra $\psi(k,\theta_m) \approx 0.3\times10^{-4}k^{-4}$, which are still the *dissipative form* of the *spectra* for enough low wave numbers, can easily give $\omega^{-4}$ form of *frequency* spectra at $^{\omega/\omega}{}_p<3$ (due to modulation by peak orbital velocities), even though the spectra range $^{\omega/\omega}{}_p<3$ is relatively unaffected by Doppler shifting with orbital velocities (this was established for $k^{-4}$, but not $k^{-7/2}$ form). *Thus* we can consider SWOP integrated (reduced) spectra $F_K$ (or $\chi(k)$), as an example of fetch limited spectra with $k^{-7/2}$ *nondissipative* part of equilibrium with the rapid transition to the *dissipation subrange* at $\hat{k}_g=3{,}2k_p$, or with $\frac{k_u U_a^2}{g} \approx 1.8$. With wind speed for SWOP 9 m/sec. the $k_g = 1.8\frac{g}{Ua}$ leads to $\frac{\omega_g U_a}{g} \approx 4$, which is in much better agreement with observations by Kitaigorodskii (1986, 1987) and Hansen et al. (1990), than the value of $\frac{k_u U_a^2}{g} \approx 2.5\text{-}3.7$, which were based on assumption that SWOP spectra is closed to *fully developed* waves, which in reality *it is* not $k_p > (2-4)\frac{g}{Ua^2}$ (see Table 1). Therefore, we can conclude that SWOP spectra if being considered as fetch growth spectra, gives *acceptable values* for transition to dissipative subrange both in *wave number* and *frequency domain*(!).

4.b. The recent stereophotogrammetric analysis (Banner et al., 1989) produce the results which according to *their authors* don't support the wave number dependence predicted by the *equilibrium spectra* for the wavelength range 0.2-1.6m, inspite of the fact that those wave lengths appear to have no preferred directions. In particular, the correlation with the wind direction is very low (only fine scale structure $\lambda < 0.2$m, seems to have much stronger visual correlation with wind direction).

Because the whole spectral range of wave lengths was not covered by stereophotoanalysis in this work, we have decided to *more carefully examine* the data reported by Banner et al. (1989), with the purpose to check the possibility to see the transition from *equilibrium* form of the spatial spectra to the *dissipation subrange* in the same way as it was done by us (Kitaigorodskii, 1984) with SWOP data. The range of key parameter $\frac{Ux^2k}{g}$ covered by Banner et al. (1989) was $(1{,}75\text{-}100)\cdot 10^3$. Wind speeds were from 5.5 to 13.3m/sec. With average value of drag coefficient $10^{-3}$, this corresponds to the range of nondimensional wave number $\frac{kU_a^2}{g} = 1.6\text{-}90$ which is *basically* the range of wave numbers on the *rear face* of relatively well developed waves ($\hat{k} > \hat{k}_p$). The Fig. 4 in Banner et al. (1989) summarize the results of *four* experiments and was interpreted as a proof of a *saturation* in this range of scales. (Within the 95 % confidence limits, there is no observational support for the linear dependence on $U_x$ implied by equilibrium spectra. While relating the wind speed at 54m (at oil platform) to the surface friction velocity $u_x$ is not straightforward, we still consider the values $\frac{k_u U_a^2}{g} \sim (2-90)$ to be reliable estimates of the conditions of the open sea wind waves, measured in Banner et al. (1989). A *closer* look at the data presented in Fig. 4 leads us to the following conclusions. In exp. 3, whose conditions are similar to Hansen et al. (1990) there is an evidence of more rapid spectral cut off at approximately $\frac{U^2K}{g} \approx 2.10^{-2}$. We would like to interpret this as a *transition* from equilibrium $k^{-7/2}$ to *dissipation subrange*. For wind speed 5,5m/sec the corresponding $U_x \approx 20\text{-}22$ cm/sec (Kitaigorodskii and Donelan, 1984), and that gives us for *transitional* wave number $\hat{k}_g$ the values $\frac{k_g U_a^2}{g} \approx 9\text{-}12$ or $\frac{\omega_g U_a}{g} \approx 3\text{-}4$ which is not far from the transitional characteristics reported by Kitaigorodskii (1987), Hansen et al. (1990).

In exp. 4 with wind speed 13, 3 m/sec the *transitional* wave number $\frac{Uxkg}{g}$ is as high as (5-6) $10^{-2}$, (this number was also picked up by us from Fig. 4 in Banner et al., 1989), which leads, with drag coefficient $1.5 \cdot 10^{-3}$, to the value $\frac{k_g U_a^2}{g} \approx 31\text{-}40$ and thus $\frac{\omega_g U_a}{g} \approx$ 5.5-6.3 which is noticeably higher than in exp. 3. However, the wave age in exp. 4 is about twice smaller than in exp. 3, which indicated that transitional wavenumber and frequency moves to lower value as waves developed in agreement with Kitaigorodskii (1983) and Hansen et al. (1991). Also dominant wave period in exp. 3 $T_d = 6.6$ sec. leads to the value $\frac{\omega_d U_a}{g} \approx 0.16 << \frac{\omega_g U_a}{g} \sim 3\text{-}4$, which means that in this case there is enough space for *equilibrium* $k^{-7/2}$ spectra still above the peak enhancement region in exp. 3. The same can be roughly said about the exp. 4, where $T_d = 5.5$ sec leads to $\omega_d \approx 1.54 \frac{g}{u_a}$ which is still at least three times smaller than the values of $\omega_g$, so *in exp. 4* there is also indirect evidence for existence an equilibrium $k^{-7/2}$ range with transition to the *dissipation* subrange. In both cases $\omega_d = \frac{2\pi}{T_d} \approx \omega_p$ (Banner, private communication).

Conditions of exp. 2 was characterized by strong winds and white capping and that's possibly why in fig. 4 of Banner et al. (1984) for experiment 2 there is no indication of *transition*, since dissipation subrange can occupy the whole range of observed wavenumbers (as well as in exp. 1). Indeed the description of experimens in Banner et al. (1989) (see Table 1 and 2) indicates that only exp. 3 and 4 corresponds to relatively *steady* conditions, and they both show some evidence of existence *transition to dissipation* subrange, which does not disagree with calculations in Hansen et al. (1990) about the movement of $\omega_g$ toward lower frequency with wave growth (either with fetch or duration).

4.c. In Phillips (1985) paper on *equilibrium spectra*, one of the sets of experimental data which were used to prove the existence of wind-dependent statistical equilibrium, where recent field measurements by Tang & Shemdin (1983) of the frequency *spectra of slope* at a fixed point. Their results clearly indicate that the frequency spectra of slope in the windward direction $S_{11}$ is proportional to friction velocity and flat, *independent of* $\omega$, provided the frequency is sufficiently smaller than those influenced by convective effects of the larger waves and currents. The Fig. 4 in Phillips (1985) of $S_{11}(\omega)$ – frequency spectra of slope in the upwind, downwind direction, clearly indicated the *lower frequency* part of $S_{11}$ ($\omega$) independent on $\omega$. However it is exactly the data on this figure, which we'll try now, as before, to interpret as *noncontroversial* evidence of *transition to dissipation subrange* in equilibrium spectra. The latter at frequencies $\omega$ between 2 rad sec.$^{-1}$ and ~ 6 rad. sec$^{-1}$ for *slope* according to (33-35) is independent on $\omega$.

$$S_{\zeta\zeta}(\omega) = S_{11}(\omega) + S_{22}(\omega) = A\,ug^{-1} \tag{39}$$

At higher frequencies where Doppler shifting effects can be significant, and interpretation is difficult, the measured spectra clearly don't follow the form (39) but decrease approximately as $\omega^{-1}$ (see Fig. 4 in Phillips (1985)), what can be interpreted as another assymptotic regime (36), corresponding to *dissipation* subrange. Over the low frequency range there is a good deal of sampling error, but not much systematic trend in the spectral levels below about 7 rad. sec$^{-1}$. The spectral densities measured at this range (39) are generally consistent with the value of Kitaigorodskii constant A, estimated in Kitaigorodskii (1983). Also Tang and Shemdin (1983) found that the downwind and transverse mean square slopes were about equal (isotropy) in cases of a wind wave field with a single well defined peak. (But the most interesting feature of Fig. 4 – the existence of *transition* to dissipation *subrange* was not ever mentioned in Phillips (1985) who argued that kinematic effects due to tidal currents and peak wave orbital velocities becomes serious at frequencies about 15 rad/sec, as well as *dynamical* limitations (capillarity and influence of drift currents). However the *transition* from frequency independent *flat* part of *slope spectra* occur in all spectra in Fig. 4 at frequencies well *below* 15 rad/sec. Here are the main characteristics of the *transition* to *dissipation subrange* derived from four curves on Fig. 4 in Phillips (1985).

*Curve 1* – $u_x \approx 11$ cm/sec and corresponding wind speed are in the range 2.2-3.3 m/sec. with value of $\omega_g = 15$ rad/sec (!) locates the *transition* to the *dissipation subrange* at $\frac{w_g U_a}{g} \approx 3.36$-5.05, which is very close to $\omega_g \approx 4 \frac{g}{U_a}$ the best estimate of $\omega_g$ according to Kitaigorodskii (1987), Hansen et al. (1990). However, we'll discuss the value $\omega_g = 15$ rad/sec in curve 1 in more detail below.

*Curve 2* – $u_x \approx 27$ cm/sec and corresponding wind speed (8.1-7.1 m/sec) with *observed* value of $\omega_g = 5.8$-$7.0$ *rad/sec* locates the *transition* to the *dissipation subrange* at $\frac{\omega_g U_a}{g} \approx 4.2$-4.8, again consistent with all previous estimates.

*Curve 3* – $u_x \approx 28$ cm/sec and corresponding wind 8.15-7.10 m/sec with *observed* value of $\omega_g = 7.0$ rad/sec$^{-1}$ locates the transition to the *dissipation* subrange at $\frac{w_g U_a}{g} \approx$ 5.0-5.82, which is a little bit higher value then the usually accepted 4.0.

*Curve 4* – $u_x \approx 45$ cm/sec and corresponding wind speed in the range 11.8-14.5 with *observed* values of $\omega_g = 5.0$-5.5 rad/sec locates the transition to the dissipation subrange at $\frac{\omega_g U_a}{g} \approx 6.0$-8.1 which is *now noticeably* higher then the results from Curves 1 and 2. To explain this *trend* in the movement of $\frac{w_g U_a}{g}$ (from curve 1) towards *high frequencies* (to curve 4) we have examined the original data by Tang and Schemdin (1983).

First of all we found that for Curve 1 the choice of $\omega_g = 15$ rad./sec. is probably not well justified. The more close to reality will be the choice of $\omega_g = 6$ rad./sec., which we'll lead to $\frac{\omega_g U_a}{g} \approx 2.14$. The latter value make the trend in $\frac{\omega_g U_a}{g}$ (from curve 1) to curve (4) even more visible. In Table 1 we present the summary of Tang and Shemdin (1983), Banner et al. (1989) and SWOP results together with results summarized in Table 1 of Hansen et al. The clear evidence of the decrease of $\omega_g$ with the movement of peak frequency region towards low frequencies is evident. This is a very intersting and important support of the ideas of Kitaigorodskii (1983) about existence of dissipation subrange in wind wave spectra.

4.d. Actually usual approach in estimating some transitional regime is limited by getting some typical average value of $\frac{\omega_g U_a}{g}$. The latter is true also for the data analysis presented in Leykin and Rozenberg (1984) which we'll discuss in 4d. We also analyzed the data of Birch, Ewing (1986). In most of cases their transitional frequency $\widetilde{\omega}_g = \frac{\omega_g U_a}{g}$ was less than $\frac{\omega_p U_a}{g}$, which makes their study not suitable for defining the dissipation subrange. By the way the old Burling (1959) data analyzed by Phillips (1958) and Kitaigorodskii (1980), even though in principle looking very convincing in terms of $\omega^{-5}$ region, *but* on average Burling (1959) data have a range of $\frac{U_a \omega_g}{g} \approx$ 0.157-0.23 being very close to $\widetilde{\omega}_p$ in their case, and thus also made them nonsuitable for determination of dissipation subrange. Even the data presented by Kimmo K. Kahma and Charles J. Calken (unpublished), which for "grand" average of dimensionless spectra give clear vision of dimensionless frequency $\frac{\omega_g U_a}{g}$ about 5 as a possible transition to $\omega^{-5}$ region, we have found not adequate for searching on dissipation subrange because for *four* (out of 7) groups of the spectra in Lake Ontario where "transition" was observed (7,6.5,3) on fig. 5 of their paper the corresponding ratios of $\frac{\omega_g}{\omega_p}$ were equal to 1.15, 1.25, 1.14, 1.2, i.e., all of them in peak enhancement region! However Fig. 6, 7 in their work is important for finding grand average value $\frac{\omega_g U_a}{g} = 5$. The frequencies below 1.2 $\omega_p$ has been excluded from the beginning and thus the grand averaged transitional value $\frac{\omega_g U_a}{g} = 5$ is reliable.

4.e. In the Leykin and Rosenberg (1984) 20 spectra was chosen to characterize the rather developed waves in the range of wind speeds 3,5-13.5′ m/sec. The measured spectra within the frequency range from 2.4 to 7.2 Hz. were analyzed with the purpose to find empirical description of the *rear faces* of the spectra. It was found that for all of the spectra in the range, $1{,}2 \leqslant \frac{\omega}{\omega_p} \leqslant 3.2$ i.e., *outside* of *peak enhancement* region, there is transition from wind dependent $\omega^{-4}$ form to $\omega^{-5}$ form of saturation. To interpret this transition as a transition to *the dissipation subrange* we recalculate below $\frac{\omega}{\omega_p}$ values into $\frac{\omega U a}{g}$ values, by assuming again $k_p = (2\text{-}4) \frac{g}{U_a^2}$. This leads to $\omega_p = (1.4\text{-}2) \frac{g}{Ua}$ and with $\omega_g \simeq 3{,}2\ \omega_p$ (Fig. 9 in Leykin and Rosenberg (1984)) to $\frac{w_g U_a}{g}$ 4.48 ~ 6.4 with average value $\frac{w_g U_a}{g} \simeq 5{,}4$, which is not to far from result of determination of the transition to the dissipation subrange reported by Hansen et al. (1990), but a little bit larger. *In this range* of frequencies, Doppler shifting by peak orbital velocities as well as permanent drift current are not enough important, so that we can consider the Leykin and Rozenberg (1984) result also as *indirect* proof of existence of the transition from nondissipative form of equilibrium $k^{-7/2}$ spectra, ($\frac{k}{k_p} \leqslant 10$), to the dissipation subrange ($\frac{k}{k_p} \geqslant 10$) with broad angular distribution. Thus the interpretation of Leykin and Rosenberg frequency spectra (1984) can be done basically in the similar way as in Hansen et al. (1990), i.e., in agreement with asymptotic predictions (33-36) of the theory of non-dissipative and dissipative parts of equilibrium spectra.

4.f. In a *recent paper* by Banner (1990) the emphasis was on the empirical *most detailed description* of *fetch* limited wave growth spectra given by Donelan et al. (1985), henceforth referred to as DHH. In latter work the rear face of frequency spectra were successfully described by $\omega^{-4}$ wind dependent (linearly) form in the range of $1.5 \leqslant \frac{\omega}{\omega_p} < 3$ i.e., *excluding the peak enhancement region* but also in the region where Doppler shifting effects by currents and peak orbital velocities are still not important. However, the DHH data which covered practically all stages of wave growth doesn't *show a transition* to *dissipation* subrange, which according to all previously analyzed material must occur at smaller than $\frac{\omega}{\omega_p} \simeq 3$ scales. This transition was not even considered by Banner (1990) and he did not interpret this region as an equilibrium of the type (33-35). That is why instead of this, Banner (1990) choose *pure empirical canonical form* of the spectra $\psi(k,\theta)$ coresponding to DHH data with angular spreading distribution for $\frac{k}{k_p} \leqslant 2.6$ (spreading cutoff doesn't occur at $\frac{k}{k_p}$ as low as $\frac{k}{k_p} \sim 2.6$ ($\frac{\omega}{\omega_p} \sim 1.6$), *extrapolated* to much *shorter* scales, consistent with the broad directional distribution observed by Banner et al. (1989). However, according to the latest views by Donelan (cited in Banner et al. 1989) see also Fig. (3a-3b) in Banner (1990), there is a *transition* to the most rapid spectrall falloff in Donelan data *(private communication)*, very similar to one observed by Longuet – Higgins et al. (1963) as a deviations from $\omega^{-4}$ form for frequencis $\omega/2\pi > 1$ HZ. In Banner (1990) this fact was not interpreted as a *transition* to the dissipation subrange (Kitaigorodskii, 1983; 1987), but rather as a direct consequence of DHH spectral form *extrapolated* to high wavenumbers with *spreading* cutoff occuring not before $\frac{k}{k_p} \geqslant 10$. However, DHH spectra in the range of, $1.5 < \frac{\omega}{\omega_p} < 3$ can be consistent with equilibrium form (33-35) with energy flux $\varepsilon o$ *being dependent on wave age* $\frac{Ua}{Cp}$ in the following way

$$\varepsilon_o = \left(\frac{cp}{U_a}\right)^{3/2} Ua^3 \; ; \; \alpha_u = \frac{S(\omega)}{g\omega^{-4} U_a} \cong 0.006 \qquad (40)$$

where in Kitaigorodskii (1983; 1986) notations

$$\varepsilon_o = mVa^3 \; ; \; m = \frac{0.006}{(2A)^3}\left(\frac{\overline{C}_p}{U_a}\right)^{3/2} \qquad (41)$$

where the Kitaigorodskii constant A is of order one (Kitaigorodskii, 1983) (A = 0.55-0.22). The fact that the energy flux $\varepsilon_o$ is *increasing* with *wave growth* ($\frac{Cp}{Ua}$) is in agreement with the results of direct calculations of $S_{nl}$ in wave generation models. Thus we can argue that DHH results together with Banner (1990) additional information are not inconsistent with *our* hypothesis about the existence of *equilibrium* form (33-35) with the *transition* to the dissipation subrange, as it was observed in Hansen et al. (1990). According to the latter work the average *value* of $\alpha_u$ in *equilibrium* spectra (33-35, 41) is equal to $4.4.10^{-3}$ which together with *observed transition* to the dissipation subrange at $\frac{\omega g Ua}{g} \simeq 2.7$ leads to the *average* value of transitional frequency .$\omega g \simeq 6.1 \frac{g}{Ua}$. This is higher than $\omega g \simeq 4 \frac{g}{Ua}$ probably due to the Doppler shifts effects, but still with observed range of $\frac{\omega p Ua}{g} \simeq$ 1-3, roughly corresponds to $\frac{\omega g}{\omega p} \geqslant 3$ in good agreement with Donelan et al. (1985).

4.g. When deriving the expression (36) corresponding to Phillips (1958) saturation form, we considered the asymptotic situation, corresponding to indefinitely large values of $\frac{k}{kg}$ (indefinitely large values of k or $\varepsilon_o$), where the statistical characteristics of the wave field are determined solely by the process of wave breaking. Therefore the magnitude of the spectrum in the dissipation subrange (36), represents according to

Phillips (1958) an *upper* limit of $F_k$, dictated by the requirement of crest attachment. generally speaking, we cannot in principle disregard the possibility (because of the very nature of asymptotic arguments) that for ($\varepsilon_o \rightarrow$ ($l \rightarrow \infty$, $k\lambda_g \rightarrow \infty$) the values of $F_k$ and $N_k$ (and therefore $S(\omega)$) continue to depend no matter how slightly on $\varepsilon_o$, so that instead of (33-36) we have

$$N_k = A\,\varepsilon_o^{1/3}\,k^{-4}\left(\frac{k}{k_g}\right)^{-p} \tag{42}$$

$$F_K = A\varepsilon_o^{1/3}\,g^{-1/2}\,k^{-7/2}\left(\frac{k}{kg}\right)^{-p} \tag{43}$$

$$S\,(\omega) = 2A\varepsilon_o^{1/3}\,g\omega^{-4}\left(\frac{\omega}{\omega_g}\right)^{-2p} \tag{44}$$

where $\omega_g$ and $k_g$ are given by expressions (32,37), and p is power exponent, such that to satisfy the predictions (35,36) it must be

$$\frac{1}{2} \geq p \geq 0 \tag{45}$$

By replacing $\varepsilon_o$ according to (40) and $k_g$ (according to (37)), (42-44) reduced to the usual wind dependent similarity form of the wind wave spectra (Kitaigorodskii (1983, 1985). Howeever, the value of p cannot be derived from dimensional considerations *only*, and the frequency spectra of the type of (44) was first analyzed by Barenblatt and Leykin (1981), who were looking for the variations of p with the stage of *wave development* (or in their terminology with the nondimensional parameter $\Lambda_o = \frac{g\lambda p}{U_a^2}$ where $\lambda p$ is peak wave length ($\lambda p = \frac{2\pi g}{\omega_p^2}$). According to their analysis (Fig. 18) the *average value of p is close to 0* (p = 0.4 ± 0.4) in the range of $\Lambda_o$ = 3-8, which corresponds to the range of $\frac{\omega p U a}{g} \approx$ 1.25-2.05. The latter is very close to *conditions* pf rather well developed waves, characterized by $k_p = (2-4)\,\frac{g}{U_a^2}$ (see section 3). Thus our interpretation of the data presented in Barenblatt and Leykin (1981) and Leykin and Rosenberg (1984) are that in the region of $1.3 < \omega/\omega p < 3$ (i.e., outside of peak enhancement region and Doppler shifting effetcts) the frequency spectra are close to the *equilibrium form* (35) ($p \approx 0$) with the possible transition *to* dissipation subrange (36), occuring only at $\omega_g \geqslant 3\omega_p$ (Leykin and Rosenberg, 1984) or approximately at $\omega_g = 3.75\text{-}6.0\,\frac{g}{U_a}$ which is close to what was observed by all others including DHH. The similar approach *was* used *for 1-D and 2-D spatial spectra* by Banner et al. (1989) whose data we have analyzed before. Banner et al. (1989) used expression for 1-D spectra of the form

$$\phi\,(k_i) \sim \left(\frac{U_*^2\,k_i}{g}\right)^{\gamma} 2k_i^{-3}\ (i = 1,2);\ (k = k_1, k_2) \tag{46}$$

where $\gamma = (1/2\text{-}p)$ was found also close to 0 (actually $\gamma = 0.09 \pm 0.09$ at the 95 % confidence limit). Thus Banner et al. (1989) confirm the asymptotic predictions corresponding to $p = 1/2$ embracing wind dependent approximation to the observed one-dimensional and two-dimensional wave number spectra, with some evidence of existence of the *transition* to the *dissipation* subrange (see sections 3,4). The most important results of data analysis we present in the Table 1, where we summarize our *determinations of the transition* to the dissipation subrange according to the different authors. The future more complete studies of the wave characteristics hopefully permit to check the results presented in Table 1.

## 5. Conclusion

We can conclude that all existing data about the frequency and spatial characteristics of wind wave field are not inconsistent with our assumption (Kitaigorodskii, 1983) about the existence of the transition to *the dissipation* subrange at high wave numbers and frequencies. Moreover, it appears that in *most cases such transition is definitely associated also with existence of equilibrium energy cascade pattern* in wind wave spectra at the scales larger than the transitional scale $\lambda_g = \frac{2\pi}{k_g}$.

TABLE 1

The summary of characteristics of the transition to the dissipation subrange in wind wave spectra

| NN | Ratio of transitional frequency $\omega_g$ to peak frequency $\omega_p$ (average value and range of variability) | Nondimensional transitional frequencies $\frac{\omega_g U_a}{g}$ (average value and range of variability) | Nondimensional peak frequency $\frac{\omega_p U_a}{g}$ (average value and range of variability) | Comments | Source |
|---|---|---|---|---|---|
| 1 | 4.8 - 5.25<br>5.0 | 6.0<br>5.76 - 6.3 | 1.20 | Run 108<br>N IV in the source | Tang & Shemdin, 1983 |
| 2 | 5.57<br>4.98 - 5.16 | 5.35 | 0.96 | Run 108<br>N III in the source | Tang & Shemdin, 1983 |
| 3 | 5.06 | 4.44 - 4.59 | 0.89 | Run 326<br>N II in the source | Tang & Shemdin, 1983 |
| 4 | 5.48 | 2.14 | 0.39 | Run 328<br>not clearly defined $\omega_g$ (in this case)<br>$\omega_g \cong 6$ rad/sec. | Tang & Shemdin, 1983 |
| 5 | 4.48 - 5.97<br>5.21 | 3.0 - 4.0<br>3.5 | 0.54 | Exp. 3, Td = Tp | Banner et al., 1989 |
| 6 | 3.59 - 4.10 | 7.0 - 8.0 | 1.54 | Exp 4<br>Td ≅ Tp in Banner et. al. | Banner et al., 1989 |
| 7 | 1.78 | 1.28 | 0.78 | See (1) below | SWOP |
| 8 | 2.85 | 7.36 | 2.58 | | Hansen et al., 1990 al 8 run 246 |
| 9 | 3.90 | 7.45 | 1.91 | | Hansen et al., 1990 run 248 |
| 10 | 3.29 | 5.73 | 1.74 | | Hansen et al., 1990 run 252 |
| 11 | 3.28 | 4.05 | 1.23 | | Hansen et al., 1990 run 254 |
| 12 | 3.2 | 4.48 - 6.4<br>5.4 | 1.25 - 2.05 | | Leykin and Rosenberg (1984) |
| | 3.0 | 3.75 - 6.0 | (1.65) | | |

(1). These numbers for $k_g = 0.2^{-1}$m ($\frac{k_g}{k_p} = 3.2$); $U_a = 9$ m/sec.; $\frac{\omega_g}{\omega_p} = 1.78$; $\frac{\omega_g U_a}{g} = 1.28$. If instead of $k_g = 0.2$ M$^{-1}$ accept P.M. fully developed, or $\omega_p = 1.4$-$2$ g/Ua, then $\omega_g = 3.56$ or 1.57, which means that $k_g = 0.2^{-1}$m is too small. For larger $k_g$, $\frac{\omega_g U_a}{g}$ will be larger than 1.28.

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Submitted to the Academy August 1991.
Published May 1992.